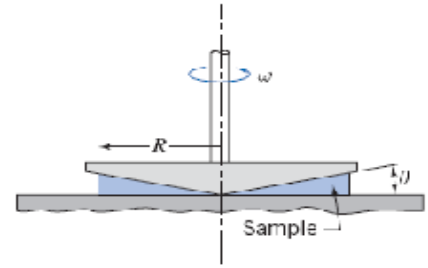


## Problem 2.68

[Difficulty: 4]

**2.68** The viscometer of Problem 2.67 is used to measure the apparent viscosity of a fluid. The data below are obtained. What kind of non-Newtonian fluid is this? Find the values of  $k$  and  $n$  used in Eqs. 2.16 and 2.17 in defining the apparent viscosity of a fluid. (Assume  $\theta$  is 0.5 degrees.) Predict the viscosity at 90 and 100 rpm.

Speed (rpm)	10	20	30	40	50	60	70	80
$\mu$ (N·s/m <sup>2</sup> )	0.121	0.139	0.153	0.159	0.172	0.172	0.183	0.185



**Given:** Data on the viscometer

**Find:** The values of coefficients  $k$  and  $n$ ; determine the kind of non-Newtonian fluid it is; estimate viscosity at 90 and 100 rpm

**Solution:**

The velocity gradient at any radius  $r$  is

$$\frac{du}{dy} = \frac{r \cdot \omega}{r \cdot \tan(\theta)}$$

where  $\omega$  (rad/s) is the angular velocity

$$\omega = \frac{2 \cdot \pi \cdot N}{60} \quad \text{where } N \text{ is the speed in rpm}$$

For small  $\theta$ ,  $\tan(\theta)$  can be replaced with  $\theta$ , so

$$\frac{du}{dy} = \frac{\omega}{\theta}$$

From Eq. 2.11,

$$k \cdot \left( \frac{du}{dy} \right)^{n-1} \frac{du}{dy} = \eta \cdot \frac{du}{dy}$$

where  $\eta$  is the apparent viscosity. Hence

$$\eta = k \cdot \left( \frac{du}{dy} \right)^{n-1} = k \cdot \left( \frac{\omega}{\theta} \right)^{n-1}$$

The data is

N (rpm)	$\mu$ (N·s/m <sup>2</sup> )
10	0.121
20	0.139
30	0.153
40	0.159
50	0.172
60	0.172
70	0.183
80	0.185

The computed data is

$\omega$ (rad/s)	$\omega/\theta$ (1/s)	$\eta$ (N·s/m <sup>2</sup> ×10 <sup>3</sup> )
1.047	120	121
2.094	240	139
3.142	360	153
4.189	480	159
5.236	600	172
6.283	720	172
7.330	840	183
8.378	960	185

From the *Trendline* analysis

$$k = 0.0449$$

$$n - 1 = 0.2068$$

$$n = 1.21$$

The fluid is dilatant

The apparent viscosities at 90 and 100 rpm can now be computed

N (rpm)	$\omega$ (rad/s)	$\omega/\theta$ (1/s)	$\eta$ (N·s/m <sup>2</sup> ×10 <sup>3</sup> )
90	9.42	1080	191
100	10.47	1200	195

